Relativistic Mean Field calculations of nuclear properties in early stages of stellar collapse.

F. K. Sutaria[a], J. A. Sheikh [a]and A. Ray [b] *

[a]Tata Institute of Fundamental Research (TIFR),
Bombay: 400 005, INDIA

[b]Code 661, Goddard Space Flight Center, Greenbelt, MD20771, USA

We use the Relativistic Mean Field (RMF) method to calculate properties of neutron rich, usually deformed nuclei, important for equation of state calculations and which have significant abundance in the early stages of stellar collapse. We compare the results of our microscopic calculations with existing cold nuclear equations of state based on macroscopic liquid drop model and the FRLDM model.

1. Introduction

Spectroscopy of neutrinos produced by electron capture on neutron-rich f-p shell nuclei having significant abundance in the cores of nearby presupernova and collapsing stars, and emitted before neutrino trapping sets in (at core density of $\simeq 10^{12}$gm/cm^3), can yield useful information on the physical conditions and nuclear composition of the core because they stream freely through the overlying stellar matter without any further interactions[1].

The neutrino spectrum depends, among other factors, on nuclear abundances. The post-silicon burning ($\rho \geq 10^9$ gm/cm and $T \geq 6 \times 10^9$ deg. K) core composition consists of neutron rich f-p shell nuclei whose abundance is controlled by nuclear statistical equilibrum. The nuclear abundances thus depend on the temperature dependant nuclear binding energies, and through the neutron and proton fractions X_n and X_p, on the neutron and proton chemical potentials μ_n and μ_p. The mass of the homologously collapsing core and the strength of the hydrodynamic shock after stellar core bounce are determined by the lepton fraction and consequently by the electron capture rates[3]. Since the e$^-$-capture threshold too, depends on the nuclear binding energies it is neccessary to calculate the chemical potentials and binding energies accurately, as functions of core temperature and density.

Because the temperature and the drip neutron fraction are relatively low, the cold nuclear equation of state (EOS) used should reproduce well the laboratory values of nuclear binding energies and chemical potentials for the neutron rich nuclei of interest. Since it must also take into account the nuclear shell and pairing effects which persist upto temperatures $\simeq$ 0.5 to 1 MeV, these quantum effects have to be calculated using microscopic mean-field Hartree Fock or RMF methods. Thus, EOS based on the classical

*NAS-NRC Senior Research Associate; on leave of absence from Tata Institute of Fundamental Research

liquid-drop models[3]−[7], can only hold accurately beyond $\rho_{trapping} \simeq 10^{12}$ gm/cm^3 and T $\simeq$ 1 MeV where nuclear shell and pairing effects are washed out. For physical conditions before neutrino trapping sets in, the cold EOS should go smoothly into the high temperature macroscopic liquid drop model based EOS[7]. Since analytical forms of EOS are less computationally expensive than detailed microscopic calculations when incorporated into hydrodynamic codes to simulate stellar collapse and explosion, we look for analytical models which would compare well with the microscopic RMF calculations.

2. Existing Equations of State

In general, all EOS developed so far approximate the ensemble of heavy nuclei and drip nucleons by a lattice consisting of a single heavy spherical nucleus (A, Z) (which is the most bound system for a given core configuration) immersed in a sea of drip neutrons of density n_n and electrons of density n_e.

In the cold, incompressible liquid drop formalism, the nuclear matter energy of a single nucleus in a lattice, W_N, is given by[2],[3] :

$$W_N(x, \rho_N, V_N, u) = W_{bulk} + 290x^2(1-x)^2A^{\frac{2}{3}} + \beta x^2\rho_N^2V_N^{\frac{5}{3}}(1 - \frac{3}{2}u^{\frac{1}{3}} + \frac{1}{2}u) \quad (1)$$

where x is the proton fraction , ρ_N the nuclear density, V_N the nuclear volume and u the fraction of total volume occupied by nuclei ($u = \rho/\rho_N$). This model accounts for the effects of drip neutrons and the nuclear lattice, but not for the nuclear deformation or shell and pairing effects. Minimising the energy of the system with respect to the nucleons bound in nuclei at fixed n_NA, n_NZ, n_n and n_NV_N gives the mass number of the (most bound) 'mean' nucleus A.

3. Results

We used the microscopic RMF approach with the Lagrangian set with non-linear self interactions for the σ-meson to calculate nuclear properties for isolated nuclei at zero temperatures, since it has been known to reproduce the ground state properties of β-stable nuclei with sufficient accuracy. The parameters that enter into the Lagrangian include the nucleon mass M_B, the masses of the σ, ω and ρ mesons (m_σ, m_ω, m_ρ) and the coupling constants g_σ g_ω and g_ρ. These are self consistently determined by variational calculation. The mesonic masses and coupling constants are treated as fixed parameters, their values having been derived by fitting to ground state properties of a few select spherical nuclei. The pairing is dealt within the BCS approximation and the initial values of the pairgaps were obtained from the odd-even mass differences, but it was found that to reproduce experimental binding energies, it was neccessary to decrease pairgaps as the nuclear assymetry increased. Fig.(1) compares the experimental binding energies with the RMF values for a range of nuclei in interest to this stage of collapse. Nuclear deformations are shown in Fig.(2) for spherical Ni and deformed Zn isotopes. The neutron chemical potential μ_n is obtained from the incompressible liquid drop model as[8]:

$$\mu_n = -16 + 125(0.5 - x) - 125(0.5 - x)^2 - 290x^2(1-x)^2A^{-\frac{1}{3}}\frac{(3-7x)}{2(1-x)} \quad (2)$$

Figure 1. Binding Energies vs. Neutron no. N for Mn, Fe, Co, Ni and Cu isotopes

Figure 2. Quadrupole(β_2) deformation vs. Neutron no. N for Ni and Zn isotopes

Figure 3. Comparision of analytical models with RMF calculations for Ni isotopes

Figure 4. Comparision of analytical models with RMF calculations for Zn isotopes

The first 3 terms in this expression come from the nuclear bulk energy, the last from coulomb and surface contributions to the nuclear energy.

Since the nuclear matter is being treated as an ensemble of isolated cold nuclei with a low drip neutron fraction, we can also derive a nuclear equation of state using the Finite Range Liquid Drop Model[9] (FRLDM). This gives the following expression for the volume contribution to μ_n :

$$\mu_n|_{vol} = -16.00 + 123.04(0.5 - x)^2 + 246.08x(0.5 - x) \tag{3}$$

and an expression similar to that in eq.(2) for the finite size and coulomb effects. Note that the bilinear term in eq.(3) reduces to the linear term in eq.(2) when nuclei in neighbourhood of $x \simeq 0.5$ are considered. The results of the FRLDM model are compared with the RMF and values from eq.(2) in Fig.(3) for the spherical Ni systems and in Fig.(4) for the deformed Zn systems. We find that the FRLDM model reproduces better the RMF values than eq.(2) even in the case of spherical Ni nuclei.

3.1. Extension of the EOS to finite temperatures.

At finite temperatures, the free energy F of the system is modified to[5]

$$F = W_N - \frac{a}{A}\frac{m^*}{m}T^2 \tag{4}$$

where a is the nuclear level density parameter, and m^* is the effective nucleon mass, which in general is a function of the nuclear temperature. We are using the RMF code to calculate the level density of the last filled orbitals to extract the coefficient of the temperature dependent correction.

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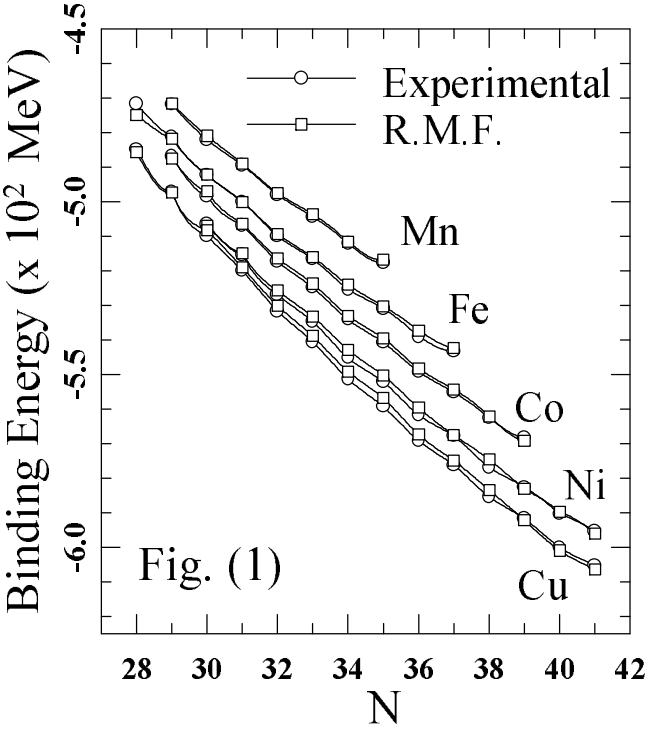
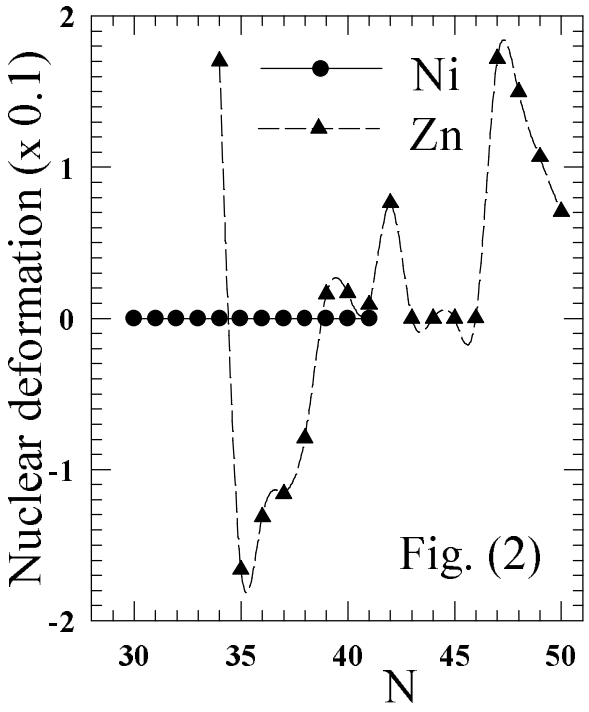
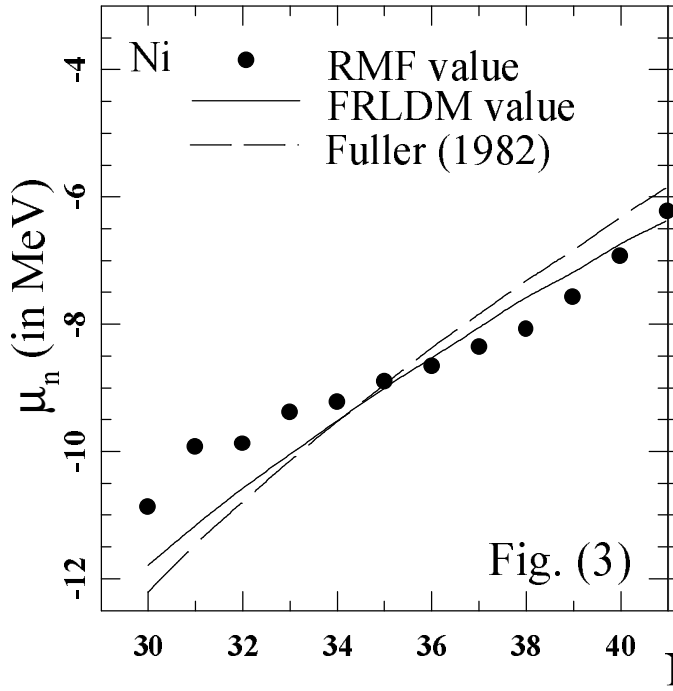
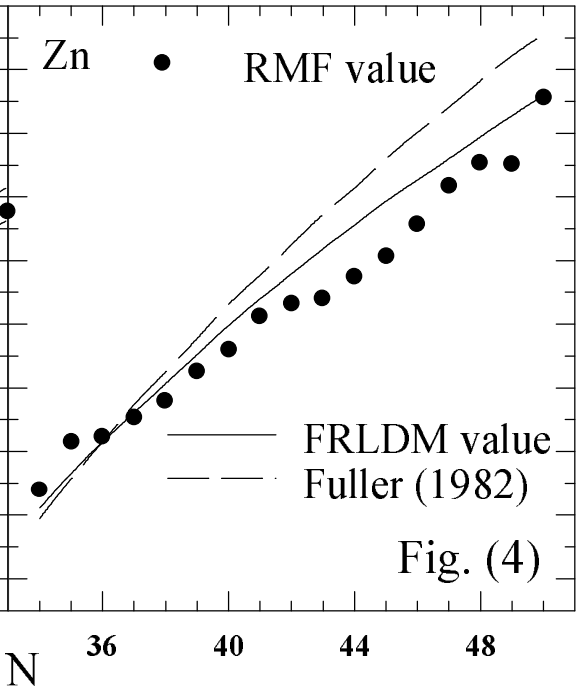

Binding Energy (x 10² MeV)
-4.5
-5.0
-5.5
-6.0
Experimental
R.M.F.
Mn
Fe
Co
Ni
Cu
Fig. (1)
28
30
32
34
36
38
40
42
N


Nuclear deformation (x 0.1)
2
1
0
-1
-2
Ni
Zn
Fig. (2)
30
35
40
45
50
N


μn (in MeV)
-4
-6
-8
-10
-12
Ni
RMF value
FRLDM value
Fuller (1982)
Fig. (3)
30
32
34
36
38
40
N


Zn
RMF value
FRLDM value
Fuller (1982)
Fig. (4)
36
40
44
48